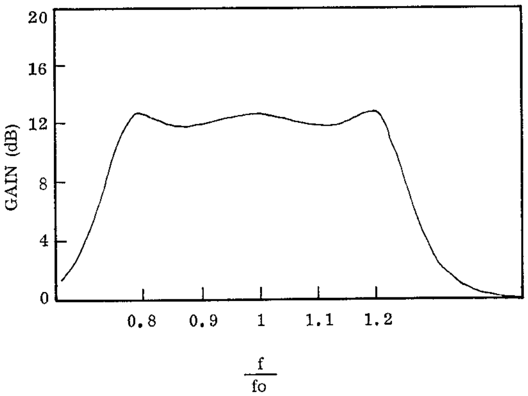

Fig. 4. Amplifier response for 40-percent 3-dB bandwidth using distributed matching network.

TABLE II
NORMALIZED PARAMETER VALUES

$Z01 = 2$	$l_1 = 0.0727$
$Z02 = 0.282$	$l_2 = 0.478$
$Z03 = 0.80$	$l_3 = 0.25$
$Z04 = 2.2$	$l_4 = 0.5$
$Z05 = 1$	$l_5 = 0.25$

values (see Table II) for 12-dB gain amplifier utilizing the distributed matching network illustrated in Fig. 1(d). The results obtained for a prescribed 3-dB bandwidth of 40 percent (±0.5-dB ripple) are presented in Fig. 4.

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Loss Calculations for Coupled Transmission-Line Structures

V. K. TRIPATHI

***Abstract*—Expressions are derived for computing loss in coupled TEM structures in terms of complex even- and odd-mode propagation constants and characteristic impedances of the lines. The attenuation due to conductor (series) and dielectric (shunt) losses in a given structure can be determined utilizing these expressions. The results may be particularly useful for computing conductor loss in microwave circuits with a large number of sections used in traveling-wave devices and as microwave circuit elements. The method is applied to some typical structures for computing conductor loss.**

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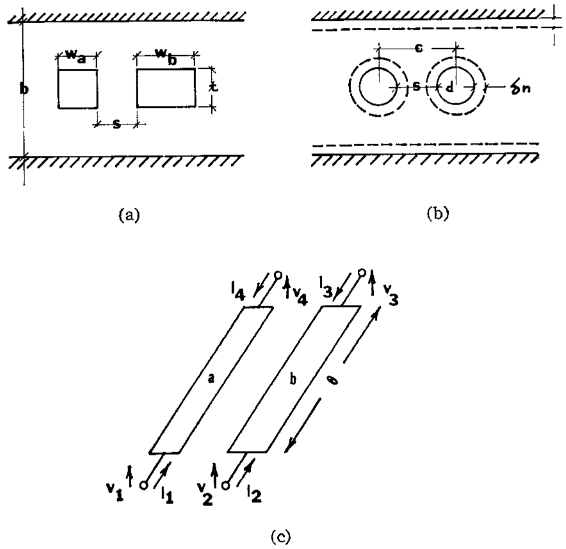

Fig. 1. (a) Rectangular coupled bars between parallel ground planes. (b) Circular coupled bar cross section showing an inward perturbation of all metal surfaces by δn. (c) Schematic of a parallel coupled line four-port.

TABLE I

UNIT SECTIONS	ATTENUATION PER SECTION, NEPERS
1- ALL PASS	$\alpha p = \dfrac{Z^r_{oe}\,\alpha_e l + Z^r_{oo}\,\alpha_o l}{Z^r_{oe} + Z^r_{oo}}$; $\beta p = \theta$
2- ALL PASS	$\alpha p = \dfrac{Y^r_{oe}\,\alpha_e l + Y^r_{oo}\,\alpha_o l}{Y^r_{oe} + Y^r_{oo}}$; $\beta p = \theta$
3- ALL PASS (Meander)	$\alpha p^* = \dfrac{\sqrt{Z^r_{oe}\,Z^r_{oo}}\,(\alpha_e l + \alpha_o l)}{Z^r_{oe}\cos^2\theta + Z^r_{oo}\sin^2\theta}$; $\beta p = \operatorname{Cos}^{-1}\left(\dfrac{Z^r_{oe} - Z^r_{oo}\tan^2\theta}{Z^r_{oe} + Z^r_{oo}\tan^2\theta}\right)$
4- BAND PASS (Interdigital)	$\alpha p^{\dagger} = \dfrac{\left[\sin\theta\,(Y^{r2}_{oo}\,\alpha_o l - Y^{r2}_{oe}\,\alpha_e l) + \left(\dfrac{1+\cos^2\theta}{\sin\theta} \mp 2\,\dfrac{\cos\theta}{\theta}\right) Y^r_{oo}\,Y^r_{oe}\,(\alpha_e l - \alpha_o l)\right]}{\sin\beta p\left[(Y^r_{oo} - Y^r_{oe})^2\right]}$ where, $\beta p = \cos^{-1}\left(\dfrac{Y^r_{oo} + Y^r_{oe}}{Y^r_{oo} - Y^r_{oe}}\cos\theta\right)$
5- BAND PASS	$\alpha p^{\dagger} = \dfrac{\left[\sin\theta\,(Z^{r2}_{oe}\,\alpha_e l - Z^{r2}_{oo}\,\alpha_o l) + \left(\dfrac{1+\cos^2\theta}{\sin\theta} \pm 2\,\dfrac{\cos\theta}{\theta}\right) Z^r_{oo}\,Z^r_{oe}(\alpha_o l - \alpha_e l)\right]}{\sin\beta p\left[(Z^r_{oe} - Z^r_{oo})^2\right]}$ where, $\beta p = \cos^{-1}\left(\dfrac{Z^r_{oe} + Z^r_{oo}}{Z^r_{oe} - Z^r_{oo}}\cos\theta\right)$
6- BAND PASS (Comb-Lines)	$\alpha p^{\dagger} = \dfrac{\left[2\,Y^r_{oo}\,Y^r_{oe}\,(\alpha_e l - \alpha_o l)\left(\dfrac{2}{\sin 2\theta} \mp \dfrac{1}{\theta}\right) + \omega C(\theta)\,(Y^r_{oo}\,\alpha_o l - Y^r_{oe}\,\alpha_e l)\left(\dfrac{1}{\cos^2\theta} \mp \dfrac{\tan\theta}{\theta}\right)\right]}{\sin\beta p\left[(Y^r_{oo} - Y^r_{oe})^2\right]}$ where, $\beta p = \cos^{-1}\left(\dfrac{\cot\theta(Y^r_{oo} + Y^r_{oe}) - \omega C(\theta)}{\cot\theta\,(Y^r_{oo} - Y^r_{oe})}\right)$

* This is the same expression as obtained by Kolker [1].
† In these expressions the upper sign is for conductor (series) loss and the lower sign is for dielectric (shunt) loss.

Losses in coupled TEM networks can be estimated utilizing a complex propagation constant and a complex characteristic impedance for even and odd mode of excitation. The complex propagation constants for even and odd modes are given by

$$\gamma_e = \alpha_e + j\beta_e \quad \text{and} \quad \gamma_o = \alpha_o + j\beta_o \tag{1}$$

where α_e and α_o are the attenuation constants and β_e and β_o are the phase constants for even and odd modes, respectively. For the case of low-loss lines considered here the attenuation constants consist of a series component due to conductor loss and a shunt component due to dielectric loss. The phase constants for even and odd modes are approximately equal for this case, that is, $\beta_e \cong \beta_o \cong \beta = \omega\sqrt{\mu\epsilon}$.

For conductor loss the expressions for α_e and α_o have been obtained by Kolker [1] and Horton [2] for the case of coupled rectangular bars and can be calculated utilizing the results of Cohn [3] and Getsinger [4], respectively. Horton's expressions which lead to more accurate results are given by

$$\alpha_e = \frac{2R_s\epsilon_r Z_{oe}}{\eta^2 b}\left[-(1-s/b)\frac{\partial C_{fe/\epsilon}'}{\partial s/b} + (1+t/b) \cdot \left(\frac{\partial C_{fe/\epsilon}'}{\partial t/b} + \frac{\partial C_{f/\epsilon}'}{\partial t/b}\right) + 2\,\frac{w/b + 1 - t/b}{(1-t/b)^2}\right] \tag{2}$$

with α_o given by the same expression as (2) with Z_{oe} and C_{fe}' replaced by Z_{oo} and C_{fo}', respectively. In (2) R_s is surface resistivity in ohms per square, $\eta = 376.7\ \Omega$ free-space wave impedance, and ϵ_r is the relative dielectric constant. These values for α's can be calculated utilizing the results of Getsinger [4] and Gupta [5] as shown by Horton [2].

Attenuation constants for the case of coupled circular bars as shown in Fig. 1(b) can be obtained using the same procedure as in Horton [2]. Utilizing the expression for the conductor attenuation constant for a transmission line as given by [6]

$$\alpha_c = \frac{R_s\sqrt{\epsilon_r}}{2\eta Z_o}\frac{\delta Z_o}{\delta n} \quad \text{nepers/unit length} \tag{3}$$

and following the method of Horton the expressions for α_o and α_e are found to be

$$\alpha_o = \frac{R_s\epsilon_r Z_{oo}}{\eta^2 b}\left[-\left(1-\frac{s}{b}\right)\frac{\partial C_{oo/\epsilon}}{\partial(s/b)} + \left(1+\frac{d}{b}\right)\frac{\partial C_{oo/\epsilon}}{\partial(d/b)}\right] \tag{4a}$$

and

$$\alpha_e = \frac{R_s\epsilon_r Z_{oe}}{\eta^2 b}\left[-\left(1-\frac{s}{b}\right)\frac{\partial C_{oe/\epsilon}}{\partial(s/b)} + \left(1+\frac{d}{b}\right)\frac{\partial C_{oe/\epsilon}}{\partial(d/b)}\right] \tag{4b}$$

where $C_{oo} = C_g + 4C_m$ and $C_{oe} = C_g$. These can be calculated utilizing the graphs for C_g/ϵ and C_m/ϵ obtained by Cristal [7].

For the case of dielectric loss the attenuation constant is proportional to the ratio of conductance to capacitance per unit length as given by $\alpha = \text{Re}\ \sqrt{j\omega L(G + j\omega C)} \simeq \frac{1}{2}G/C\sqrt{LC}$. It is the same for even and odd modes since for both modes of excitation $G/C = \sigma/\epsilon$. Therefore,

$$\alpha_o = \alpha_e = \alpha_d = \frac{\pi\sqrt{\epsilon_r}}{\lambda}\tan\delta \tag{5}$$

where $\tan\delta$ is the loss tangent of the dielectric, λ is the wavelength, and ϵ_r is the relative dielectric constant.

The characteristic even- and odd-mode impedances of the lines are also complex and are, approximately, given by

$$Z_{oo} \cong Z_{oo}^r\left[1 \pm j\frac{\alpha_o}{\beta}\right]$$

$$Z_{oe} \cong Z_{oe}^r\left[1 \pm j\frac{\alpha_e}{\beta}\right] \tag{6}$$

where

$$Z_{oo}^r \cong \frac{\eta\epsilon}{\sqrt{\epsilon_r}C_{oo}}$$

$$Z_{oe}^r \cong \frac{\eta\epsilon}{\sqrt{\epsilon_r}C_{oe}}$$

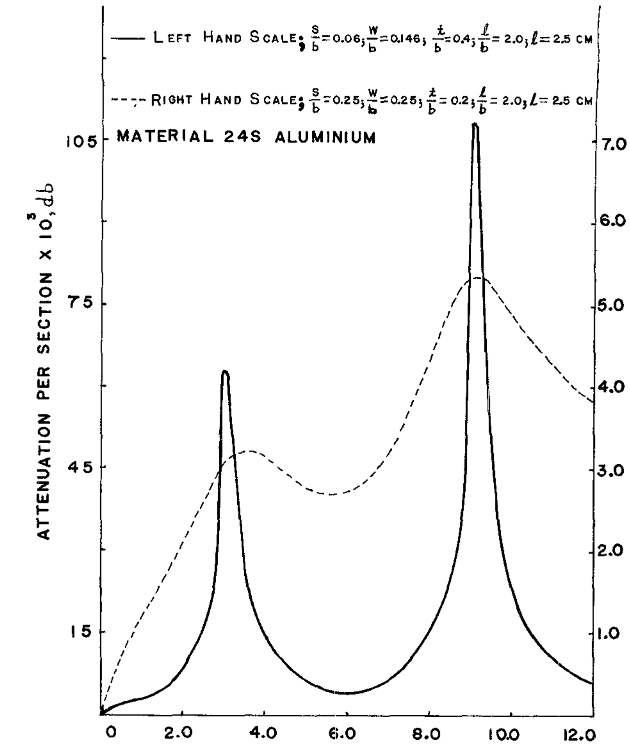

Fig. 2. Attenuation response of meander lines (Case 3; Table I).

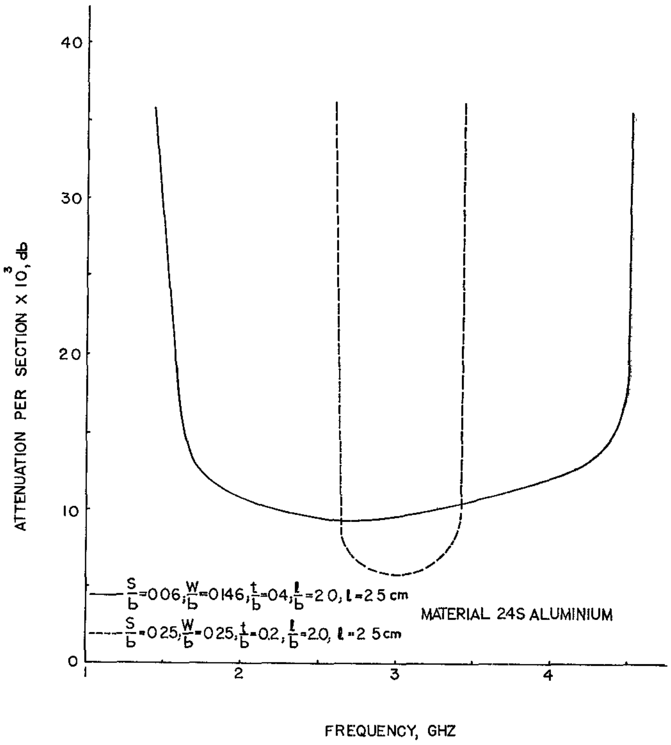

Fig. 3. Attenuation response of interdigital lines (Case 4; Table I).

and $\beta = \omega\sqrt{\mu\epsilon}$. The plus sign is for series (conductor) loss and minus sign for shunt (dielectric) loss.

The immittance parameters for two coupled-line four-ports as shown in Fig. 1(c) can be obtained by proceeding as in Jones and Bolljahn [8] postulating four current or voltage sources and utilizing the expressions for propagation constants and characteristic impedances as given by (1) and (6), respectively. For the case of low-

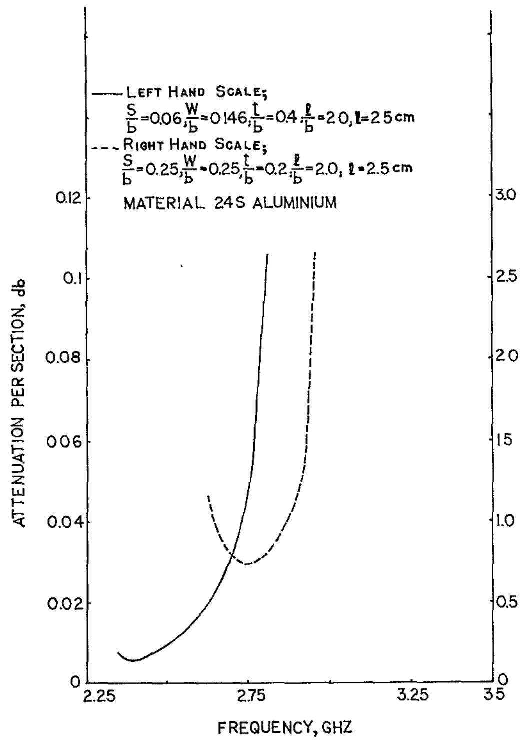

Fig. 4. Attenuation response of comb-line structures (Case 6; Table I with $C(\theta)=1$ pF).

loss identical lines the impedance parameters are found to be

$$Z_{11} = Z_{22} = Z_{33} = Z_{44} = \tfrac{1}{2}[-j \cot\theta(Z_{oe} + Z_{oo}) + \csc^2\theta(Z_{oe}{}^r\alpha_e l + Z_{oo}{}^r\alpha_o l)] \quad (7a)$$

$$Z_{12} = Z_{21} = Z_{34} = Z_{43} = \tfrac{1}{2}[-j \cot\theta(Z_{oe} - Z_{oo}) + \csc^2\theta(Z_{oe}{}^r\alpha_e l - Z_{oo}{}^r\alpha_o l)] \quad (7b)$$

$$Z_{14} = Z_{23} = Z_{41} = Z_{32} = \frac{-j \csc\theta}{2}[(Z_{oe} + Z_{oo}) + j \cot\theta(Z_{oe}{}^r\alpha_e l + Z_{oo}{}^r\alpha_o l)] \quad (7c)$$

$$Z_{13} = Z_{31} = Z_{24} = Z_{42} = \frac{-j \csc\theta}{2}[Z_{oe} - Z_{oo}) + j \cot\theta(Z_{oe}{}^r\alpha_e l - Z_{oo}{}^r\alpha_o l)] \quad (7d)$$

where $\theta=\beta l$. The admittance parameters are determined in a similar manner utilizing voltage sources.

Losses in a given structure may then be calculated in terms of loss due to individual sections as a function of frequency. Table I shows the expressions for attenuation per section for some typical unit filter sections consisting of coupled lines with various boundary conditions existing at individual ports of the structures.

The conductor loss per section as a function of frequency of some useful slow wave structures (filters) is plotted in Figs. 2, 3, and 4 for some typical values of structure dimensions. These are calculated utilizing the expressions given in Table I and the graphs for C_{fe}/ϵ', C_{fo}/ϵ', and C_{f}/ϵ' obtained by Getsinger [4] and Gupta [5] for coupled rectangular bars.

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Computer-Aided Renormalized Perturbation Method for the Inhomogeneously Loaded Waveguide

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***Abstract*—The renormalized perturbation method is applied to the inhomogeneously loaded waveguide. The second-order term in the usual Rayleigh–Schrödinger perturbation method is divergent with respect to increasing the concerned mode number. Introducing the phenomenological parameter by analogy to the quantum electrodynamics, we have the nondivergent second-order perturbation term.**

The Rayleigh–Ritz variational method and the finite-difference solution method are adequate to the eigenvalue problems of the inhomogeneously loaded waveguide, as shown in Fig. 1 [1]–[4]. Consumed computing time should be proportional to N^3 for the solution of the $N \times N$ matrix in these procedures (N is the concerned mode number in the R–R method, and in the finite-difference method, it is the total number of mesh points [4]). Since the computing time accompanying the large N is very long, we may request a more simple method for it.

The perturbation method has been believed to be applicable only to problems that are very similar to exactly solvable problems [1]. The usual Rayleigh–Schrödinger perturbation method includes some difficulties. The greatest difficulty is that the second-order (and/or higher order) terms are divergent with respect to increasing the concerned mode number, even if the loading is weak [7]. In this correspondence the extensive perturbation method, which excludes the divergence, will be given.

The Rayleigh–Schrödinger perturbation equation is described as [5], [6]

$$\gamma_i = \gamma_i{}^0 + \Gamma_{ii} + \sum_{j\neq i}^{\infty} \frac{\Gamma_{ij}\cdot\Gamma_{ji}}{\gamma_i{}^0 - \gamma_j{}^0} + \cdots \quad (1)$$

where $\gamma_i{}^0$ and $\gamma_j{}^0$ are the propagation constants of the i and j mode of the unperturbed waveguide, respectively, and Γ_{ij} is the perturbation Hamiltonian

$$\Gamma_{ij} = \langle\Psi_i{}^0 | L | \Psi_j{}^0\rangle \quad (2)$$

where $|\Psi_i{}^0\rangle$ shows the normalized unperturbed eigenfunction of the i mode, and L is the perturbation operator [5], [6].

We discuss a thin resistive film loaded waveguide as shown in Fig. 1. In Fig. 1, if $1/R\cdot\sqrt{(\mu_0/\epsilon_0)}\cdot(d/b)$ is constant with respect to the change of d/b, then the first-order perturbation term in (1) is constant, and thus we call it the constant loading. Physically, it is expected that, according to $d/b\to 0$ (i.e., $R\to 0$) under the constant loading, the perturbed propagation constant γ_i goes near to $\gamma_i{}^0$, because the $d/b=0$ load gives no perturbation for the electromagnetic wave, even if the load has high conductivity.

The result of Rayleigh–Schrödinger calculation for constant loading is shown in Fig. 1. At small d/b, i.e., small R, the calculated propagation constant is divergent. This situation is the same as the

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